

10. Homework Assignment
Dynamical Systems II

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<http://dynamics.mi.fu-berlin.de/lectures/>
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Problem 1: Consider the system of differential equations

$$\begin{aligned}\dot{x} &= xy, \\ \dot{y} &= -y + x^3.\end{aligned}$$

Use a (local) center manifold to decide whether the equilibrium $x = y = 0$ is asymptotically stable.

Note: Use the invariance of the center manifold to calculate the necessary terms of its Taylor expansion.

Problem 2: Determine the center manifold of the equation

$$\begin{aligned}\dot{x} &= \mu x - x^2, \\ \dot{y} &= y - x^2, \\ \dot{\mu} &= 0.\end{aligned}$$

Is the center manifold of the equilibrium at the origin smooth?

Problem 3: Can a flow possess transverse homoclinic or heteroclinic points to:

- (i) stationary points?
- (ii) periodic orbits?

Problem 4: Let Φ be a diffeomorphism of the plane \mathbb{R}^2 with a transverse homoclinic orbit. In class, we found a shift on only two symbols for an iterate Φ^n . Prove that for every $m \in \mathbb{N}$ the shift of m symbols is conjugate to some iterate Φ^n on a suitable subset of \mathbb{R}^2 .